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Regression methodology for sensitivity analysis of solar heating walls

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Abstract

Passive solar heating of buildings continues to be a great interest of renewable energy applications. Part of this interest focuses on solar heating walls. A solar heating wall (SHW) is a part of building walls that receive, store, and transfer solar thermal energy into the building. SHW sensitivity analysis is often performed to guide optimum designs. For the sensitivity analysis, the methodology used so far is numerical simulations. This work presents a new approach, regression analysis, and develops a general regression model. To show how to develop a specific model for a given SHW from the general one, how to do the regression, and how to validate the model, the lattice solar heating wall (LSHW) is selected as a case study. Detail heat transfer analysis is performed to develop the specific regression model for the LSHW sensitivity analysis. Four side-by-side test cells are constructed to obtain experimental data. The data are then used to determine the regression constants and coefficients and the time series numbers. Validation of the regression analysis shows that the model has very high confidence. The model is also used for LSHW optimization, yielding the same results as those from the simulation with the numerical simulation program, which further demonstrates that the proposed model is reliable.

Keywords: Regression; Solar heating wall; Sensitivity analysis

1. Introduction

Passive solar heating of buildings is a great interest of renewable energy applications. Part of this interest focuses on the thermal performance study of new materials and configurations of solar heating walls [1–5].

The solar heating wall (SHW) is a part of building walls that is designed to receive, store, and transfer solar thermal energy into the building. SHW examples are Trombe walls [6–8], phase-change-material solar walls [9,10], lattice solar walls [11], composite wall solar collectors [2], and honeycomb insulation walls [12,13].

Factors that influence SHW thermal performances can be classified as two categories: (a) design parameters, such as shading, orientation, insulation, glazing, wall configurations, and thermal properties, and (b) climate conditions, mainly solar radiation and ambient temperature.

To analyze the influence of design parameter changes on SHW thermal performances, sensitivity analysis is performed [14,15]. When analyzing the influence of a chosen design parameter on SHW thermal performances, the sensitivity analysis is performed by hour-by-hour calculations in a given climate pattern, allowing only this design parameter to change while keeping the rest constant.

To the author's best knowledge, methodology used so far for SHW sensitivity analysis is numerical simulations. Experimental measurements are only used to validate the simulation program.

Regression analysis, a powerful tool for solar energy applications [16–19], is scarcely used for SHW study. There are only two papers found out in the author's extensive literature search [20,21], and both of them are only for SHW thermal performance study.

The methodology of regression is especially useful for the study of new SHW materials and/or configurations where numerical solutions are not available in a period of time. The objectives of this study are: (a) to propose a general regression model for SHW sensitivity analysis; (b)

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Nomenclature

A_0	regression constant (bias)
a_j, b_k	regression coefficients
c_{lj}, d_{lk}	regression coefficients ($l = 1, 2, 3, 4$)
D	hydraulic diameter of the LSHW vent
D'_{h}	the ratio of LSHW vent height to wall height
D_{t}	LSHW thickness
I	solar radiations
(i)	at time i
M, N	time series numbers to be determined by regression analysis
M_l, N_l	time series numbers to be determined by regression analysis ($l = 0, \ldots, 4$)
Po	porosity of a lattice wall (vent section area to total wall area)
Q	solar heating wall heat gain
Q_{vc}	LSHW heat gain through vent air circulation
Q_{vr}	LSHW heat gain through vent radiation heat transfer
Q_{wc}	LSHW heat gain through inside surface convective heat transfer
Q_{wr}	LSHW heat gain through inside surface radiation heat transfer
T_{a}	ambient temperature
$T_{l\mathrm{v}}$	LSHW vent surface temperature
$T_{l\mathrm{w}}$	LSHW inside surface temperature
T_{r}	room temperature
T_{v}	average temperature of circulation air at the point leaving vent to room
T_{w}	average temperature of test cell inside surfaces, except for LSHW inside surface

to take the lattice solar heating wall (LSHW) as an example to show how to develop a specific model from the general one, how to arrange test matrix, and how to perform regression analysis; and (c) to validate the concretized model by comparing the predicted values with experimental measurements and those obtained from numerical simulation program.

2. A general regression model for SHW sensitivity analysis

Jiménez and Heras [20] and Kennish et al. [21] all assumed that the SHW is a linear system. The model Kennish et al. [21] proposed is, in the present symbols, of the form:

$$Q(i) = \sum_{j=0}^{M} a_j[T_{\mathrm{a}}(i-j) - T_{\mathrm{r}}(i-j)] + \sum_{k=0}^{N} b_k I(i-k) \qquad (1)$$

This model cannot be used for SHW sensitivity analysis because all design parameters "hide" in regression constant regression coefficients a_j and b_k, and therefore cannot be seen. For SHW sensitivity analysis, the design parameters to be analyzed must appear in the regression equation.

The correctness of Eq. (1) is obvious from the theory of transfer function [22]. However, it is not practical even for SHW thermal performance study because its inputs include room temperature that is also an output needing to be determined.

From the point of view of heat transfer, a building system with SHWs can be approximated to a linear system. Consider this system as a "black-box" with multiple inputs and single output. The inputs are the solar radiation and ambient temperature, and the output is the room temperature. Accordingly, the room temperature is the linear superposition of the solar radiation and ambient temperature. Therefore, Eq. (1) can be rewritten as

$$Q(i) = A_0 + \sum_{j=0}^{M} a_j T_{\mathrm{a}}(i-j) + \sum_{k=0}^{N} b_k I(i-k) \qquad (2)$$

Eq. (2) applies when the following requirement meets:

(a) During experimental measurements, the room temperature is uncontrolled and there is no inner heating source; or
(b) During experimental measurements, the room temperature is controlled to a preset point, with little fluctuation.

Consider Z is a regression coefficient. It can be expressed as

$$Z = Z_0 + Z_1 f \qquad (3)$$

where Z_0 and Z_1 are constants, and f is a function of the design parameters. Consequently, Eq. (2) can be rewritten as

$$Q(i) = A_0 + \sum_{j=0}^{M_0} a_j T_{\mathrm{a}}(i-j) + \sum_{k=0}^{N_0} b_k I(i-k) + \sum_{j=0}^{M_1} f c_j T_{\mathrm{a}}(i-j) + \sum_{k=0}^{N_1} f d_k I(i-k) \qquad (4)$$

Eq. (4) is a general regression model for SHW sensitivity analysis. Function f can be determined with the following two approaches:

(a) Heat transfer analysis; and/or
(b) Multi-order approximation. For example, if x and y are two design parameters to be analyzed, the f may be assumed to be

$$f = a_1 x + a_2 x^2 + a_3 y + a_4 y^2 + a_5 xy + a_6 x^2 y + a_7 xy^2 + \cdots \qquad (5)$$

where $a_i(i = 1, 2, 3, \ldots)$ are some constants. It can be seen that this method may yield more regression coefficients, which will lower the regression confidence.

3. A methodology using heat transfer analysis to determine function *f*

The method starts from analyzing the SHW heat gain to find out the related design parameters. The lattice solar heating wall (Fig. 1) is selected to demonstrate how to use this method. The heat gain Q from the lattice solar heating wall (LSHW) is

$$Q = Q_{\mathrm{wr}} + Q_{\mathrm{wc}} + Q_{\mathrm{vr}} + Q_{\mathrm{vc}} \quad (6)$$

In Eq. (6), the radiation heat transfer between the glazing inner surface and room surface is omitted because it is much smaller compared with any item included in Eq. (6).

To simplify the example, assume all LSHWs are constructed with the same materials, brick for instance, so that the thermal properties of the walls are not to be considered in the heat transfer analysis. Also, assume that the wall height is unchanged. Under the above assumptions,

$$Q_{\mathrm{wr}} \propto (1 - \mathrm{Po})(T_{l\mathrm{w}}^4 - T_{\mathrm{w}}^4) \quad (7)$$

$$Q_{\mathrm{wc}} \propto (1 - \mathrm{Po})(T_{l\mathrm{w}} - T_{\mathrm{r}}) \quad (8)$$

$$Q_{\mathrm{vr}} \propto \mathrm{Po}(T_{l\mathrm{v}}^4 - T_{\mathrm{w}}^4) \quad (9)$$

and

$$Q_{\mathrm{vc}} \propto \frac{\mathrm{Po}}{D'_{\mathrm{h}}\sqrt{1 - D'_{\mathrm{h}}}}(T_{\mathrm{v}} - T_{\mathrm{r}}) + A_1\left(\frac{D_{\mathrm{t}}}{D^2\sqrt{D'_{\mathrm{h}}\sqrt{1 - D'_{\mathrm{h}}}}}\right)^{2/3}(T_{l\mathrm{v}} - T_{\mathrm{v}}) \quad (10)$$

where A_1 is a constant. In Eqs. (7)–(10), temperatures $T_{l\mathrm{w}}$ and $T_{l\mathrm{v}}$ are related to the LSHW heat capacity, i.e., $(1-\mathrm{Po})D_{\mathrm{t}}$, when the thermal properties are not considered. Based on the above discussion, it follows:

$$f = \left\{\mathrm{Po}, \frac{\mathrm{Po}}{D'_{\mathrm{h}}\sqrt{1 - D'_{\mathrm{h}}}}, \left(\frac{D_{\mathrm{t}}}{D^2\sqrt{D'_{\mathrm{h}}\sqrt{1 - D'_{\mathrm{h}}}}}\right)^{2/3}, (1 - \mathrm{Po})D_{\mathrm{t}}\right\} \quad (11)$$

Combining Eq. (4) with Eq. (11) yields the following regression model for LSHWs:

$$\begin{aligned} Q(i) = {} & A_0 + \sum_{j=0}^{M_0} a_j T_{\mathrm{a}}(i-j) + \sum_{k=0}^{N_0} b_k I(i-k) \\ & + \mathrm{Po}\left[\sum_{j=0}^{M_1} c_{1j} T_{\mathrm{a}}(i-j) + \sum_{k=0}^{N_1} d_{1k} I(i-k)\right] \\ & + \left(\frac{D_{\mathrm{t}}}{D^2\sqrt{D'_{\mathrm{h}}\sqrt{1 - D'_{\mathrm{h}}}}}\right)^{2/3} \\ & \times \left[\sum_{j=0}^{M_2} c_{2j} T_{\mathrm{a}}(i-j) + \sum_{k=0}^{N_2} d_{2k} I(i-k)\right] + \frac{\mathrm{Po}}{D'_{\mathrm{h}}\sqrt{1 - D'_{\mathrm{h}}}} \\ & \times \left[\sum_{j=0}^{M_3} c_{3j} T_{\mathrm{a}}(i-j) + \sum_{k=0}^{N_3} d_{3k} I(i-k)\right] + (1 - \mathrm{Po})D_{\mathrm{t}} \\ & \times \left[\sum_{j=0}^{M_4} c_{4j} T_{\mathrm{a}}(i-j) + \sum_{k=0}^{N_4} d_{4k} I(i-k)\right] \end{aligned} \quad (12)$$

4. Experimental facilities and test matrix

The hot box method is well-known for testing building components. The American Society of Testing and Materials proposed the guarded hot box configuration [23]. According to [23] a metering chamber width of 0.82–1.22 m is practical for most types of panels.

Four side-by-side test cells, in light of the ASTM C236 requirements, have been built to conduct LSHW experiments (Fig. 2). The four test cells have the same inner dimension of $H \times W \times L = 0.96 \times 0.8 \times 1.2$ m, and each test cell has a different LSHW configuration. The east and west outside walls, floors and roofs are constructed with 150 mm

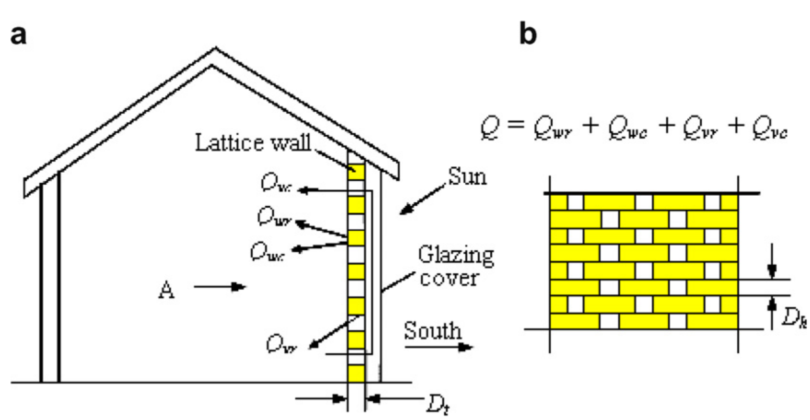

Fig. 1. Lattice wall and lattice wall heated building. (a) Lattice solar wall heated building, (b) lattice wall from view A.

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Fig. 2. Four side-by-side test cells.

polyfoam, and the back walls (north walls) with 20 mm polyfoam. The LSHWs face south, each with a double-glazing cover 50 mm ahead. The temperatures inside the cells are uncontrolled, and there are no inner heat sources.

When doing regression, the cases-to-independent variables (IVs) ratio should ideally be 20:1, namely 20 cases for every IV in the model. The lowest ratio should be 5:1, i.e., 5 cases for every IV in the model. In the regression of SHW sensitivity analysis, a case, or data case, is a group of data that occur at a single instance in one test cell. Eq. (4), the general regression model for SHW sensitivity analysis, is a time series regression model, in which the IVs are $T_a(i-j)$ and $I(i-k)$. Assuming weather is a periodic function in day cycle, the j and k are estimated at 24 (24 h a day). Therefore, at least 120 cases are required, and 480 cases are ideal. Furthermore, data cases from a test cell were at least 24 h in series, with the first temperature and last temperature of the SHW close in order to eliminate uncertainty caused by energy stored by the SHW. For obtaining cases in at least 24-hour series, it is advisable that an experimental period lasts at least 50 h, with measuring performed hour-by-hour.

LSHWs tested were constructed with bricks. Eq. (11) shows that in the given wall material, the LSHW f function is dependent on four structural parameters, i.e. LSHW porosity Po, LSHW thickness D_t, LSHW vent hydraulic diameter D, and the ratio of LSHW vent height to wall height D'_h. The tested range of a parameter should be wide enough to cover its optimum value. Take LSHW thickness D_t for example. In engineering practice, a brick wall may be 120, 240, 370, or 490 mm thick. The optimum thickness of a brick Trombe wall is 240 mm [24]. The optimum thickness of a LSHW should not be less than that of a Trombe wall because a LSHW has less mass compared with a Trombe wall of the same dimension. Therefore, three wall thicknesses of 240, 370, and 490 mm were arranged in the test matrix.

The experiments were conducted during typical winter time from late November to early March in Beijing, China, whose latitude is 40° North. During measuring periods, daily average ambient temperatures were between −7.5 and 8.9 °C, with daily temperature differences between the maximums and minimums around 14 °C, daily total insolation on test cell glazing were between 7.4 and 13 GJ, and daily average test cell temperatures were between 14.8 and 18.9 °C, with daily temperature differences between the maximums and minimums around between 10.5 and 21.1 °C. Two solarimeters were used to measure total insolation on test cell glazing and horizontal diffuse insolation, respectively.

Totally more than 500 data cases were obtained, among which 285 valid cases were chosen for the regression.

5. Regression results and discussion

The 285 valid cases mentioned above were used to determine the regression constants and coefficients and the time series numbers M_l and N_l ($l = 0, \ldots, 4$) in Eq. (12).

Because of the time series nature of the regression, the significance level of individual time series numbers M_l and N_l are tested instead of individual regression coefficients. The resulted correlation for LSHWs is assessed on the basis of error analysis by determining SE, the standard error between the calculated and experimental values of Q, and R^2, the coefficient of determination. Take M_3 for example. Assuming initially $M_3 = 6$, if increasing M_3 to 7 improves SE and R^2, set $M_3 = 8$, and continue the test until the best result is achieved. On the contrary, if $M_3 = 7$ makes SE and R^2 become worse, set $M_3 = 5$, and continue the test until the best result is achieved.

SE is determined by

$$\mathrm{SE} = \frac{1}{n-1}\sqrt{\sum_{i=1}^{n}\left[Q(i)_{\mathrm{exp}} - Q(i)_{\mathrm{cal}}\right]^2} \tag{13}$$

where n is the number of total cases, and $Q(i)_{\mathrm{exp}}$ and $Q(i)_{\mathrm{cal}}$ are, respectively, the experimental and calculated values of $Q(i)$.

The SE computed using Eq. (13) is 0.0286. The coefficient of determination R^2 equals 0.992, indicating that there is a very good agreement between the calculated and experimental $Q(i)$. Fig. 3 illustrates the conclusion graphically.

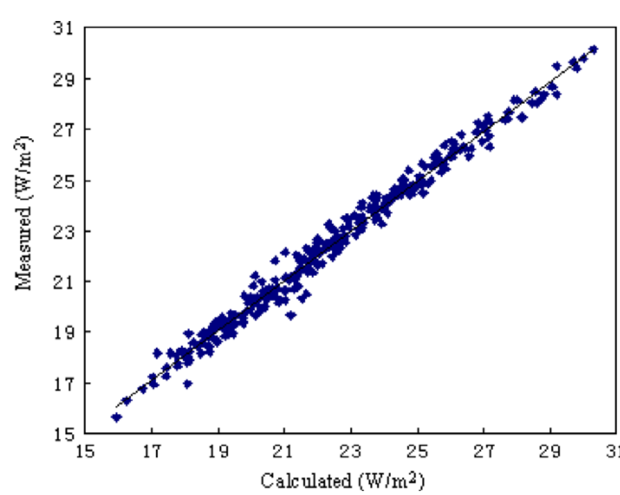

Fig. 3. Solar heating wall heat gain: measured vs. calculated.

Comparing the $Q(i)_{cal}$ with $Q(i)_{exp}$ one-by-one finds out that among all the 285 cases, the maximum error is less than 7.2%, and the number of cases that have errors great than 3.5% are less than 3.9%.

The resulted correlation was used for LSHW optimization, which yields the same results as those from the simulation with the numerical simulation program as stated by Fang and Li [11].

6. Conclusions

The only methodology used for SHW sensitivity analysis is numerical simulations. The present work proposes a new approach, regression analysis. A general regression model is developed. To validate the model and to show how to concretize the general regression model for a given SHW, the LSHW is selected as a case study. Detailed heat transfer analysis for developing the specific regression model for LSHW sensitivity analysis is conducted.

The four side-by-side test cells were constructed to conduct LSHW experiments. With hour-by-hour measurements, more than 500 data cases were obtained, among which 285 valid cases were chosen for the regression to determine the regression constants and coefficients and the time series numbers in the specific LSHW regression model.

The regression result has very high confidence. The standard error is 0.0286, and the coefficient of determination, $R^2 = 0.992$, indicating that the calculated results agree with the measurements very well. The resulted correlation is also used for LSHW optimization, yielding the same results as those from the numerical simulation program, which further demonstrated that the general regression model proposed is reliable.

The process using the regression methodology can be summarized as the following:

(a) Build test cells with the size as proposed by ASTM C236 [23] or other equivalent guidelines. Using more test cells causes higher initial cost but saves experimental time.
(b) Set text matrix based on the *f* function and winter weather conditions.
(c) Conduct experiments following the text matrix to obtain the data cases for determining the regression constants and coefficients and the time series numbers *M*'s and *N*'s. An experimental period lasts at least 50-h, measuring both cell temperatures and ambient conditions hour-by-hour. More than 250 data cases should be taken down in order to obtain at least 120 valid cases.
(d) Perform regression analysis to determine the regression constants and coefficients and the time series numbers *M*'s and *N*'s, as the example described in Section 5.
(e) Then extrapolate the results to a full scale system that operates in a different climate but within the ambient condition ranges the experiments covered. This allows performing SHW sensitivity analysis.

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