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Numerical computation of time lags and decrement factors for different building materials

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Abstract

In this study, time lags and decrement factors for different building materials have been investigated numerically. For this purpose, one dimensional transient heat conduction equation was solved using the Crank–Nicolson scheme under convection boundary conditions. To the outer surface of the wall, periodic boundary conditions were applied. Twenty-six different building materials were selected for analysis. The computations were repeated for eight different thickness of each material and the effects of thickness and the type of material on time lag and decrement factor were investigated. It was found that thickness of material and the type of the material have a very profound effect on the time lag and decrement factor. The results of present study are useful for designing more effective passive solar buildings and other related areas.
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Keywords: Building materials; Time lag; Decrement factor; Passive solar building

1. Introduction

For passive solar buildings, heating the building is possible via the direct heat gain and/or thermal storage method and there have been many researches in this area [1–3]. Although the direct heat gain method is simple and inexpensive, it suffers from large temperature swings besides strong directional day lighting [4]. In addition, the direct heat gain method can be affected very fast from outside temperature fluctuations which results in a bed comfort level for indoors [5–7]. For thermal storage buildings on the other hand, walls and floors are used as heat storage elements, and stored energy in the walls and floors during the day period can be used for heating during nights.

At the cross-section of the outer wall of a building, there are different temperature profiles during any instant of a 1-day period. These profiles are function of inside temperature, outside temperature and materials of the wall layers. Since the outside temperature changes periodically during a 1-day period, there will be new temperature profiles at any instant of time of the day. During this transient process, a heat wave flows through the wall from outside to inside and the amplitude of these waves shows the temperature magnitudes, and wavelength of the waves shows the time. The amplitude of the heat wave on the outer surface of the wall is based on solar radiation and convection in between the outer surface of the wall and ambient air. During the propagation of this heat wave through the wall, its amplitude will decrease depending on the material and the thickness of the wall. When this wave reaches the inner surface, it will have an amplitude which is considerably smaller than the value it hat at the outer surface. The times it takes for the heat wave to propagate from the outer surface to the inner surface is named as ‘‘time lag’’ and the decreasing ratio of its amplitude during this process is named as ‘‘decrement factor’’ [8]. Time lag and decrement factor are very important characteristics to determine the heat storage capabilities of any materials. Depending on the material

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and thickness of the wall, different time lags and decrement factors can be obtained. Recently conducted studies by the present author [9–11], different aspects of the time lag and decrement factor for building walls have been discussed.

In the present study, time lags and decrement factors for real building materials have been investigated numerically. For this purpose, one-dimensional transient heat conduction equation was solved for a wall under periodic convection boundary conditions. Twenty-six different building materials were selected for analysis. The computations were repeated for eight different thickness of each material and the effects of thickness and the type of material on time lag and decrement factor were investigated. In was found that different materials result in different time lags and decrement factors. In addition, it was found that the thickness of the material is very deterministic from the time lag and decrement factor point of view. The results of present study are useful for designing more effective passive solar buildings and other related energy saving areas.

2. Time lag ϕ, decrement factor f and sol–air temperature t_{sa}

Time lag and decrement factor are very important characteristics to determine the heat storage capabilities of any material. As mentioned before, the time it takes for the heat wave to propagate from the outer surface to the inner surface is named as "time lag" and the decreasing ratio of its amplitude during is named as "decrement factor". The schematics of time lag and decrement factor are shown in Fig. 1.

In this study, the time lag and decrement factor are computed as follows. The time lag is defined as

$$\phi = \begin{cases} t_{T_o^{max}} > t_{T_e^{max}} \Rightarrow t_{T_o^{max}} - t_{T_e^{max}}, \\ t_{T_o^{max}} < t_{T_e^{max}} \Rightarrow t_{T_o^{max}} - t_{T_e^{max}} + P, \\ t_{T_o^{max}} = t_{T_e^{max}} \Rightarrow P, \end{cases} \tag{1}$$

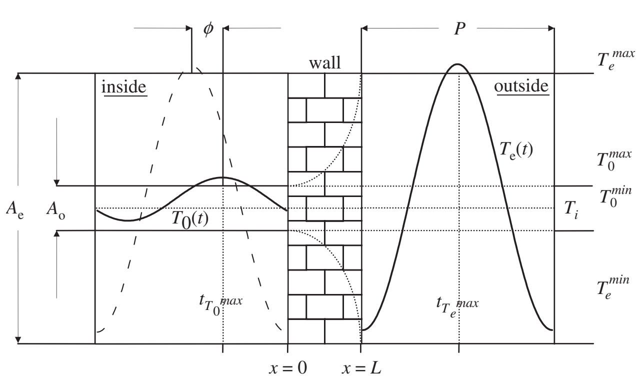

Fig. 1. The schematic representation of time lag ϕ and decrement factor f.

where $t_{T_o^{max}}$ and $t_{T_e^{max}}$ (h) represent the time in hours when inside and outside surface temperatures are at their maximums, respectively, and P (24 h) is the period of the wave.

The decrement factor is defined as

$$f = \frac{A_o}{A_e} = \frac{T_o^{max} - T_o^{min}}{T_e^{max} - T_e^{min}}, \tag{2}$$

where A_o and A_e are the amplitudes of the wave in the inner and outer surfaces of the wall, respectively.

The sol–air temperature, T_{sa}, includes the effects of the solar radiation combined with outside air temperature and changes periodically. This temperature is assumed to show sinusoidal variations during a 24-h period. Since time lag and decrement factor are dependent on only wall material, not the climatological data [12], a very general equation for sol–air temperature is taken as follows:

$$T_{sa}(t) = \frac{|T_{max} - T_{min}|}{2} \sin\left(\frac{2\pi t}{P} - \frac{\pi}{2}\right) + \frac{|T_{max} - T_{min}|}{2} + T_{min}. \tag{3}$$

3. Method

In this study, the wall under investigation is assumed to be only in the x direction and time dependent. The problem geometry is shown in Fig. 2. The one-dimensional, transient heat conduction equation for this problem is as follows:

$$k\frac{\partial^2 T}{\partial x^2} = \rho c_P \frac{\partial T}{\partial t}, \tag{4}$$

where k is the thermal conductivity, ρ is the density and c_P is the specific heat of the wall material. To solve this

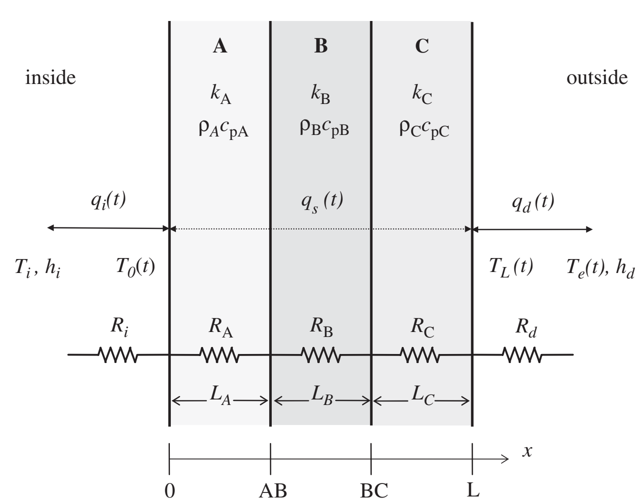

Fig. 2. The schematic of the problem geometry.

problem, two boundary conditions and one initial condition are needed. On both sides of wall, convection boundary conditions are present. At the inner surface, the boundary condition is

$$k\left(\frac{\partial T}{\partial x}\right)_{x=0} = h_{\mathrm{i}}[T_{x=0}(t) - T_{\mathrm{i}}], \tag{5}$$

whereas on the outer surface of the wall, the boundary condition can be written as

$$k\left(\frac{\partial T}{\partial x}\right)_{x=L} = h_{\mathrm{o}}[T_{\mathrm{sa}}(t) - T_{x=L}(t)]. \tag{6}$$

Here, h_{i} is the wall inner surface heat transfer coefficient, h_{o} the wall outer surface heat transfer coefficient, $T_{x=0}$ is the wall inner surface temperature, $T_{x=L}$ is the wall outer surface temperature, T_{i} is the room temperature and $T_{\mathrm{sa}}(t)$ is the "sol–air temperature".

As an initial condition, the steady-state solution of the problem at $t=0$ is taken. In the computations, the inside temperature of a room, T_{i}, is taken to be constant. As seen from Eq. (3), $T_{\mathrm{sa}}(t)$ changes in between $T_{\max}$ and $T_{\min}$ during the 24-h period. The problem now is reduced to one-dimensional heat conduction, which has a periodic boundary condition on the outer surface, the sol–air temperature boundary condition, and normal convection boundary condition on the inner surface. The analytical solution of this problem for one layer is given in Ref. [5]. Here, the algorithm is developed to take care of n layers. For this purpose, finite-difference formulation of Eq. (4) is obtained and the Crank–Nisolson method is applied. The input values of code are number of layers, the thickness of each layer, density of each layer, specific heat and conductivity of each layer and heat generation of each layer, if any. To interpret the graphics and the results better, $T_{\min} = 0\,^{\circ}\mathrm{C}$ and $T_{\max} = 1\,^{\circ}\mathrm{C}$ were selected in Eq. (3), and indoor temperature was selected as $0.5\,^{\circ}\mathrm{C}$, accordingly. The outputs of the code were time lag, decrement factor, wall inner surface temperature and the temperature of any location at any time of the day. To test the correctness of the code developed, computed time-dependent heat fluxes across the wall were compared with harmonic analysis results of Threlkeld [12] in Fig. 3. As seen from Fig. 3, computed results of the present study match pretty well with harmonic analysis results of Threlkeld. The details of the analytical solution for one layer with real climatological data can be found in the work of Threlkeld [12].

4. Results and discussion

Before we proceed with the discussion of results, let us look into materials used for computations. The materials used are given in Fig. 4 with their respective heat

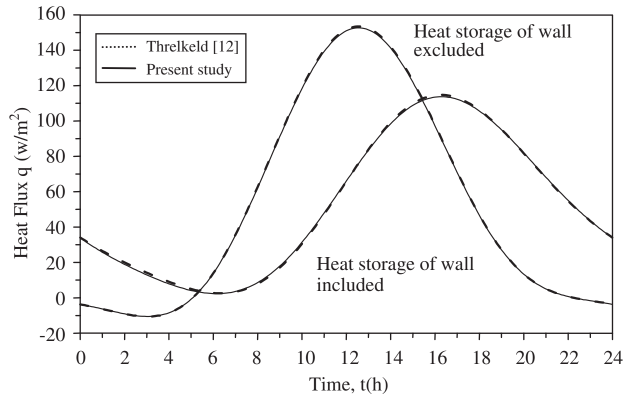

Fig. 3. Comparison of present computations with analytical solution of Threlkeld [5].

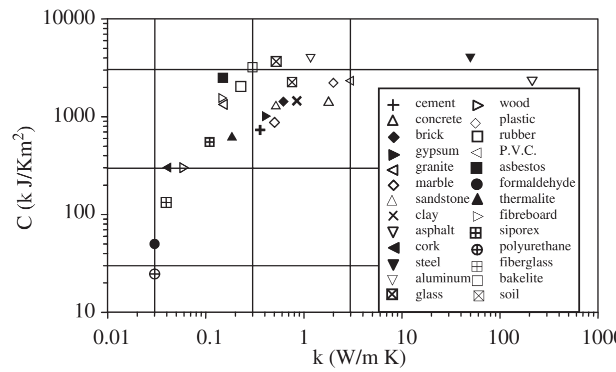

Fig. 4. Heat capacity–thermal conductivity map of building materials.

capacity and thermal conductivity values. As seen from Fig. 4, it is possible to categorize the materials into three groups. The first group is the materials with low thermal conductivity and low heat capacity like polyurethane, formaldehyde, and fiberglass. The second group is the materials with high thermal conductivity and high heat capacity like steel and aluminum. The third is the materials with moderate thermal conductivity and high heat capacity like rubber, brick, clay, and wood.

In Fig. 5a, the decrement factors vs. heat diffusivity results are presented for different thickness. Here, the materials are marked for the wall thickness of $L = 0.20\,\mathrm{m}$.

To keep the figure understandable, the materials are not marked for the other wall thickness. Again, in Fig. 5b, the time lags vs. heat diffusivity results are presented for different wall thickness. Again, the materials are marked only for the wall thickness of $L = 0.20\,\mathrm{m}$. As seen from Figs. 5a, increasing the wall thickness causes a decrease in decrement factor for all materials. For small wall thickness ($L < 0.050\,\mathrm{m}$), all the materials give almost constant decrement factors. For the wall thickness of $L \geqslant 0.050\,\mathrm{m}$, materials with high thermal diffusivity such as polyurethane and granite

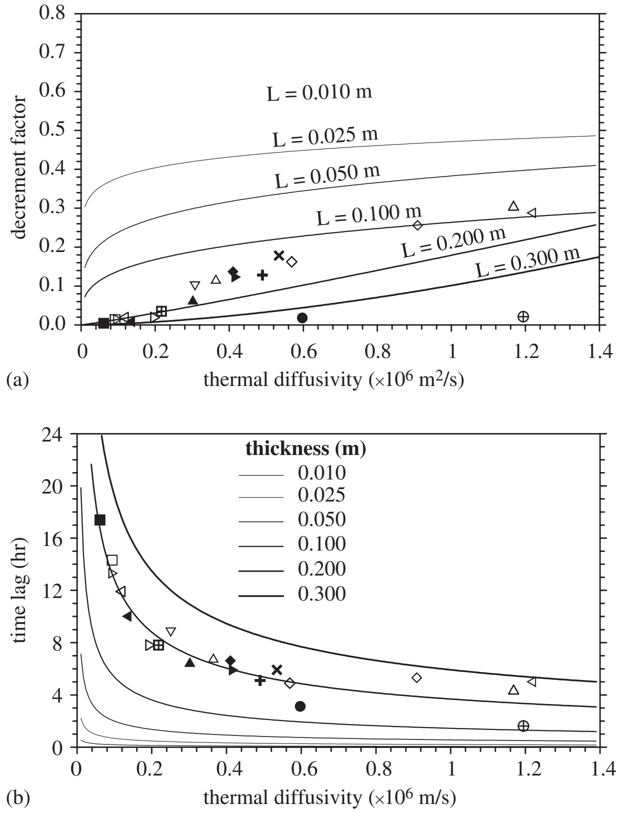

Fig. 5. (a) Decrement factor vs. thermal diffusivity of some building materials for different thickness (the position of materials for $L = 0.20\,\mathrm{m}$). (b) Time lag vs. thermal diffusivity of some building materials for different thickness (the position of materials for $L = 0.20\,\mathrm{m}$).

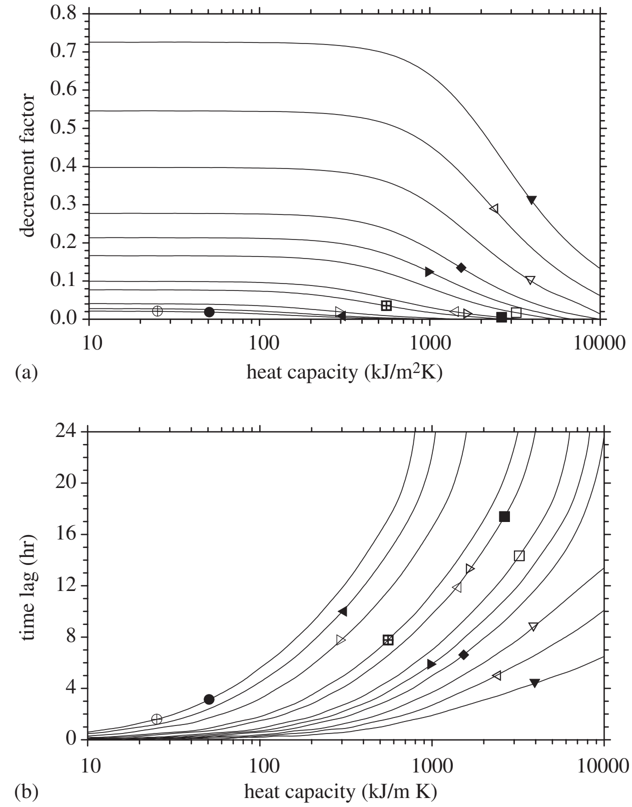

Fig. 6. (a) Decrement factor vs. heat capacity of some building materials ($L = 0.20\,\mathrm{m}$). (b) Time lag vs. heat capacity of some building materials ($L = 0.20\,\mathrm{m}$).

give considerable higher decrement factors than materials with small thermal diffusivity such as brick and concrete. So if $L \geqslant 0.050\,\mathrm{m}$, the type of material matters from the decrement factor point of view. Looking at Fig. 5b, we see that increasing the wall thickness causes an increase in time lag for all materials. For small wall thickness ($L < 0.050\,\mathrm{m}$), all the materials give almost constant the time lags. For the wall thickness of $L \geqslant 0.050\,\mathrm{m}$, materials with high thermal diffusivity such as polyurethane and granite give considerable lower time lags than materials with small thermal diffusivity such as brick, and concrete.

In Fig. 6a, decrement factor vs. heat capacity and in Fig. 6b, time lag vs. heat capacity results are given for a 0.2 m thick wall. In each figure, the lines are constant conductivity lines. As seen in Fig. 6a, the materials with heat capacity less than $700\,\mathrm{kJ/m^2\,K}$ give almost constant decrement factor. Again, an increase in thermal conductivity results in an increase in decrement factor. Again, as seen from Fig. 6a, materials with heat capacity greater than $1000\,\mathrm{kJ/m^2\,K}$ such as steel, aluminum, granite results in a sharp drop in decrement factor. From Fig. 6a, we can conclude that for the heat capacity $C \leqslant 700\,\mathrm{kJ/m^2\,K}$, the dominant factor affecting the decrement factor is the conductivity of the material. For the heat capacity $C > 700\,\mathrm{kJ/m^2\,K}$, the effect of conductivity on decrement factor starts to decrease whereas from this point on the effect of heat capacity starts to dominate. If we look at Fig. 6b, both heat capacity and thermal conductivity are equally important on time lag results. Here, materials with high thermal conductivity and heat capacity such as steel produce small time lags. We get the same results from the materials with small heat capacity and small thermal conductivity such as formaldehyde foam.

In Fig. 7a, decrement factor vs. wall thickness are given for different building materials. As seen from Fig. 7a, as the wall thickness increases, decrement factor decreases for all building materials. For materials with small thermal conductivities, such as formaldehyde foam and polyurethane foam, the decrease in decrement factor is linear in character. On the other hand, materials with high thermal conductivity such as steel, the decrease in decrement factor starts after a certain wall thickness of around 10 cm wall thickness. After this point the decrease in decrement factor is sharper than materials with small thermal conductivity. For the wall thickness of $L \geqslant 30\,\mathrm{cm}$, the decrement factor goes to zero

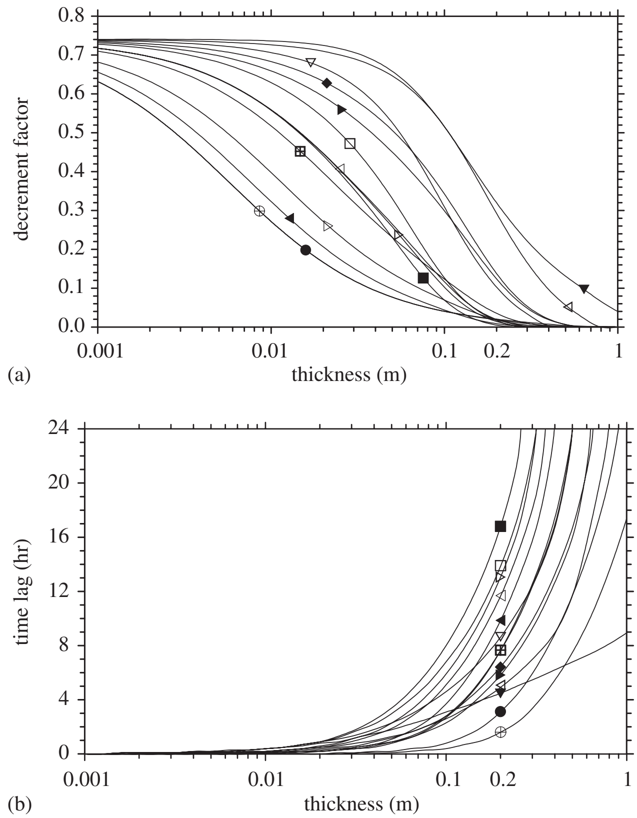

Fig. 7. (a) Decrement factor vs. thickness of some building. (b) Time lag vs. thickness of some building materials.

for materials with small and moderate thermal conductivities. For materials with high thermal conductivities zero decrement factor happens for thicker walls. Ultimately, the decrement factor goes to zero for all materials for $L \geqslant 1$ m. In Fig. 7b, time lag vs. wall thickness results are presented for different building materials. As seen from Fig. 7b, up to a certain wall thickness ($L \leqslant 1$ cm), all building materials give almost zero time lag. This means that the sinusoidal heat wave on the outer wall surface propagates inside the building without any delay. On increasing the wall thickness beyond $L > 1$ cm, time lag values start to differ for various building materials. Materials with small heat capacity and small thermal conductivity such as polyurethane foam and formaldehyde foam sustain small time lag values up to 10 cm wall thickness. From this point on, the time lag values start to increase sharply for these materials. Time lag values, for materials with high heat capacity and small thermal conductivity such as asbestos and rubber, start to increase for $L > 1$ cm. As seen from Fig. 7b, materials with high heat capacity and high thermal conductivity such as steel, gives considerable lower time lag values for all wall thickness.

Finally, in Table 1, computed time lags and decrement factors for different building materials are presented. Here, for each material, computations were performed for wall thickness of $L = 0.001$, 0.0100, 0.025, 0.050, 0.100, 0.200, 0.300, 1.000 m.

Table 1

Thickness	$L = 0.001$ m		$L = 0.010$ m		$L = 0.025$ m		$L = 0.050$ m		$L = 0.100$ m		$L = 0.200$ m		$L = 0.300$ m		$L = 1.000$ m	
Building material	ϕ (h)	f	ϕ (h)	f	ϕ (h)	f	ϕ (h)	f	ϕ (h)	f	ϕ (h)	f	ϕ (h)	f	ϕ (h)	f
Cement layer	0.01	0.730	0.09	0.647	0.26	0.544	0.69	0.426	1.89	0.284	5.12	0.128	8.23	0.054	>24	≈0
Concrete block	0.01	0.733	0.16	0.672	0.44	0.588	1.14	0.477	2.88	0.312	6.81	0.118	10.31	0.043	>24	≈0
Brick block	0.01	0.735	0.17	0.683	0.46	0.609	1.15	0.506	2.83	0.343	6.65	0.137	9.86	0.053	>24	≈0
Gypsum plastering	0.01	0.732	0.12	0.660	0.28	0.564	0.89	0.450	2.34	0.299	5.93	0.123	9.27	0.048	>24	≈0
Granite (red) block	0.02	0.739	0.24	0.726	0.59	0.701	1.28	0.646	2.62	0.515	5.01	0.288	6.95	0.166	>24	≈0
Marble (white) block	0.02	0.739	0.22	0.721	0.56	0.689	1.25	0.626	2.66	0.487	5.31	0.255	7.56	0.136	>24	≈0
Sandstone block	0.02	0.737	0.16	0.720	0.40	0.688	0.92	0.633	2.03	0.519	4.47	0.306	6.45	0.176	21.77	≈0
Clay layer	0.02	0.736	0.17	0.698	0.45	0.639	1.10	0.551	2.61	0.396	5.98	0.178	8.84	0.078	>24	≈0
Soil layer	0.03	0.732	0.40	0.669	1.31	0.569	2.93	0.409	6.12	0.184	12.08	0.036	18.65	0.001	>24	≈0
Asphalt layer	0.04	0.738	0.41	0.706	1.03	0.647	2.31	0.526	4.62	0.309	8.82	0.100	12.00	0.034	>24	≈0
Steel slab	0.04	0.741	0.38	0.736	0.89	0.719	1.79	0.658	3.05	0.516	4.41	0.313	5.09	0.227	8.95	0.031
Aluminum slab	0.02	0.741	0.23	0.739	0.55	0.733	1.13	0.708	2.09	0.631	3.43	0.459	4.14	0.352	5.86	0.113
Cork board	0.00	0.656	0.08	0.323	0.32	0.174	1.10	0.097	3.66	0.044	10.02	0.008	15.77	0.001	>24	≈0
Wood board	0.02	0.717	0.24	0.559	0.79	0.403	2.27	0.259	5.89	0.103	13.31	0.014	20.28	0.000	>24	≈0
Glass block	0.02	0.735	0.39	0.692	0.73	0.624	1.64	0.517	3.77	0.329	7.74	0.116	11.65	0.041	>24	≈0
Plastic board	0.01	0.733	0.10	0.671	0.27	0.587	0.73	0.482	1.90	0.339	4.94	0.162	7.84	0.073	>24	≈0
Bakelite board	0.01	0.724	0.34	0.603	0.96	0.466	2.32	0.315	5.76	0.136	12.53	0.022	19.49	0.001	>24	≈0
Rubber board	0.03	0.728	0.39	0.629	1.17	0.501	3.01	0.331	6.76	0.127	14.34	0.017	21.82	0.000	>24	≈0
PVC board	0.01	0.717	0.20	0.559	0.65	0.406	1.90	0.265	5.11	0.116	11.92	0.019	18.01	0.002	>24	≈0
Asbestos layer	0.03	0.716	0.37	0.557	1.23	0.396	3.39	0.230	7.97	0.069	17.41	0.004	>24	0.000	>24	≈0
Formaldehyde board	0.00	0.632	0.01	0.271	0.06	0.139	0.23	0.077	0.84	0.040	3.19	0.018	5.96	0.008	>24	≈0
Fiberglass	0.00	0.656	0.01	0.322	0.10	0.174	0.52	0.099	1.71	0.051	5.70	0.018	9.92	0.006	>24	≈0
Thermalite board	0.01	0.721	0.09	0.582	0.28	0.439	0.81	0.309	2.36	0.181	6.52	0.064	10.43	0.021	>24	≈0
Fiberboard layer	0.00	0.682	0.06	0.379	0.24	0.234	0.80	0.138	2.66	0.069	7.86	0.019	12.54	0.005	>24	≈0
Siporex board	0.01	0.710	0.09	0.517	0.26	0.355	0.92	0.231	2.81	0.123	7.81	0.035	12.31	0.009	>24	≈0
Polyurethane board	0.00	0.632	0.01	0.271	0.03	0.139	0.12	0.077	0.42	0.040	1.63	0.020	3.36	0.120	17.31	≈0

5. Conclusions

In this study, time lags and decrement factors for real building materials have been investigated. Twenty-six different building materials were selected for analysis. The computations were repeated for eight different thickness of each material and the effects of thickness and the type of material on time lag and decrement factor were investigated. It was found that different materials result different time lags and decrement factors. In addition, it was found that the thickness of the material is very deterministic from the time lag and decrement factor point of view. The results of present study are useful for designing more effective passive solar buildings and other related energy saving areas.

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