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Improved energy management method for auxiliary electrical energy saving in a passive-solar-heated residence

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Abstract

This paper describes the optimal control of operation of the auxiliary heating system used as backup in a passive-solar-heated system. A specific application is studied for a residence heated using a combined sunspace Trombe–Wall passive system solar system and an electrically heated thermal storage floor to satisfy the heating requirements of the house. The mathematical model of the general solution is derived and the optimal control determination using the maximum principle technique is described. The present work is a continuation of previous work done by Bakos [Energy Build. 31 (2000) 237]. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Auxiliary heating system; Optimal control; Passive-solar-heated system; Electrical energy saving

1. Introduction

Greece's building sector is one of the largest energy consumers taking into consideration the mild climatic conditions. Concerning the energy consumption, heating accounts the largest share in the residential/tertiary sector (60.9 and 52.5%, respectively).

In passive-solar-heated buildings, because the energy storage is often thermally coupled to the living space, the discharge from storage is determined by the governing heat-transfer equations and cannot be switched on or off. The operation of an energy storage system such as the one described requires an energy management strategy which depends on the characteristics of the system in question. An effective control strategy for energy movement must take into account the time delay between energy input and release. Several control strategies can be devised to satisfy basic requirements such as minimum cost and/or comfort [2].

In this paper, a control strategy, known as maximum principle technique which is also referred to as the quadratic objective function, will be described and its performance characteristics in a combined sunspace Trombe–Wall passive solar system will be determined. Such a design is incorporated in a house built in the city of Xanthi in northern Greece.

2. Building energy balance and modelling

The sunspace, the storage wall, the enclosure and the storage floor are modelled using a single node for each (i.e. lumped capacitance, Fig. 1) [1]. The collector-storage wall operates as a passive component by transmitting a portion of the absorbed solar energy into the building via either of the two paths. Along the first path, energy is conducted through the wall and subsequently convected and radiated from the inside wall surface into the building. The second path is convection of energy from the outer wall surface to air in the gap between the wall and the innermost glazing. This air is circulated through the gap, heated and returned to the building [3]. This model will be used to develop and analyse the minimum cost control strategy.

In the model in Fig. 1, the heat transfer is given by:

$$Q_s = H_t(\tau\alpha)A_{glaz} \quad (1)$$

$$Q_{s,loss} = U_{sa}A_{glaz}(T_s - T_a) \quad (2)$$

$$Q_{fs,loss} = U_{fa}A_{fs}(T_{fs} - T_a) \quad (3)$$

$$Q'_s = U_{sw}A_w(T_s - T_w) \quad (4)$$

$$Q_w = U_{we}A_w(T_w - T_e) \quad (5)$$

$$Q_{w,loss} = U_{ws}A_w(T_w - T_s) \quad (6)$$

$$Q_c = 2mC_p(T_m - T_e) \quad (7)$$

$$Q_f = U_{fe}A_f(T_f - T_e) \quad (8)$$

$$Q_{f,loss} = U_{fg}A_f(T_f - T_g) \quad (9)$$

$$Q_{e,loss} = UA_e(T_e - T_a) \quad (10)$$

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Nomenclature

A_f	enclosure floor area (m^2)
A_{fs}	sunspace floor area (m^2)
A_{glaz}	glazing area (m^2)
A_w	wall area (m^2)
C_e	thermal capacitance of the enclosure (J/°C)
C_f	thermal capacitance of the floor (J/°C)
C_s	thermal capacitance of the sunspace (J/°C)
C_w	thermal capacitance of the wall (J/°C)
H_t	solar radiation incident on the glazing surface (W/m^2)
T_a	ambient temperature (°C)
T_e	enclosure temperature (°C)
T_{fs}	sunspace floor temperature (°C)
T_g	ground temperature (°C)
T_m	mean temperature between innermost glazing and enclosure (°C)
T_s	sunspace temperature (°C)
T_w	wall temperature (°C)
U_{fa}	overall heat-transfer coefficient, floor to ambient (W/m^2 °C)
U_{fe}	overall heat-transfer coefficient, floor to enclosure (W/m^2 °C)
U_{fg}	overall heat-transfer coefficient, floor to ground (W/m^2 °C)
U_{sa}	overall heat-transfer coefficient, sunspace to ambient (W/m^2 °C)
U_{sw}	overall heat-transfer coefficient, sunspace to wall (W/m^2 °C)
U_{we}	overall heat-transfer coefficient, wall to enclosure (W/m^2 °C)
U_{ws}	overall heat-transfer coefficient, wall to sunspace (W/m^2 °C)
UA_e	overall enclosure heat-transfer coefficient, area product (W/°C)
$(\tau\alpha)$	glazing transmittance–absorbance product

Performing an energy balance on each node in Fig. 1, and considering that at any instant of time, the sum of the energy entering the system minus the sum of the energy leaving the system must be equal to the rate of change of energy storage in the system:

$$\text{node sunspace}: \quad Q_s - Q_{s,loss} - Q_{fs,loss} - Q'_s = C_s\frac{dT_s}{dt}$$

$$\text{node wall}: \quad Q'_s - Q_{w,loss} - Q_c - Q_w = C_w\frac{dT_w}{dt}$$

$$\text{node floor}: \quad Q_{aux} - Q_f - Q_{f,loss} = C_f\frac{dT_f}{dt}$$

$$\text{node enclosure}: \quad Q_w + Q_c - Q_f - Q_{f,loss} - Q_{e,loss} = C_e\frac{dT_e}{dt}$$

and using the above relationships:

$$g_1 = \frac{dT_s}{dt} = a_1H_t - (a_2+a_3)T_s + (a_2+a_4)T_a - a_4T_{fs} + a_3T_w \tag{11}$$

$$g_2 = \frac{dT_w}{dt} = a_5T_s - (a_5+a_6)T_w + (a_7+a_6)T_e - a_7T_m \tag{12}$$

$$g_3 = \frac{dT_f}{dt} = \frac{Q_{aux}}{CC_f} - (a_8+a_9)T_f + a_8T_e + a_9T_g \tag{13}$$

$$g_4 = \frac{dT_e}{dt} = a_{10}T_w - (a_{10}+a_{11}+a_{12}+a_{13})T_e + a_{11}T_m + (a_{12}-a_{14})T_f + a_{14}T_g \tag{14}$$

and

$$a_1 = \frac{(\tau\alpha)A_{glaz}}{C_s}, \qquad a_2 = \frac{U_{sa}A_{glaz}}{C_s}, \qquad a_3\frac{U_{sw}A_w}{C_s}$$

$$a_4 = \frac{U_{fa}A_{fs}}{C_s}, \qquad a_5 = \frac{2U_{sw}A_w}{C_w}, \qquad a_6 = \frac{U_{we}A_w}{C_w}$$

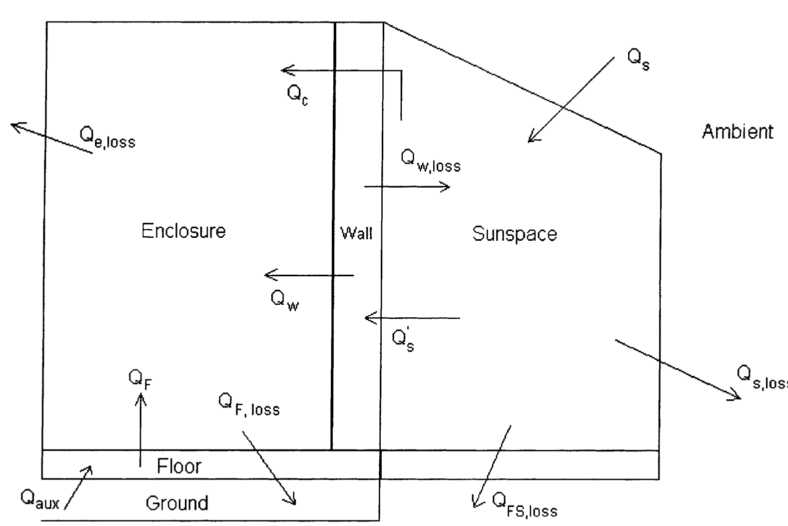

Fig. 1. Schematic of passive-solar residence.

$$a_7 = \frac{1}{2mC_pC_w}, \qquad a_8 = \frac{U_{fe}A_f}{C_f}, \qquad a_9 = \frac{U_{fg}A_f}{C_f}$$

$$a10 = \frac{U_{we}A_w}{C_e}, \qquad a_{11} = \frac{1}{2mC_pC_e}, \qquad a_{12} = \frac{U_{fe}A_f}{C_e}$$

$$a_{13} = \frac{UA_e}{C_e}, \qquad a_{14} = \frac{U_{fg}A_f}{C_e}$$

Eqs. (11)–(14) describe the thermal performance of the system shown in Fig. 1. Although the short-term performance of the system is important from a human comfort standpoint, it is the long-term performance characteristics that are important from an energy and economic standpoint [4,5]. A more detailed model could easily be developed but the one described here is sufficient for the following analysis.

3. Optimal control determination

The optimal use of auxiliary heat input can be determined using a dynamic optimization technique. The technique used in this paper is the maximum principle [6]. The statement of what is to be optimized is described from the quadratic objective function. This statement is as follows:

$$J = \int_{t_0}^{t_f} fQ_{aux}^2 + C(T_e - T_{set})^2 \, dt$$

where f represents the utility rate structure, C a comfort weighting coefficient, T_{set} is the desired temperature, all of which may be functions of time of day. The first term represents a measure of energy cost and the second is a measure of discomfort. Because Q_{aux} is squared in the first term, it is not strictly an energy-cost term but is, in fact, the product of energy cost, $fQ_{aux}\,dt$ and power demand, Q_{aux}. The objective function is minimized on a daily basis although, ideally, the objective function should be optimized over the entire heating season. The optimization over the entire heating season is more complex since the enclosure temperature will change very often around the desired temperature. As an approximation to this seasonal optimization, the objective function is minimized on a daily basis and the enclosure temperature is forced to equal the desired temperature at midnight. However, the minimization of J on a daily basis is considered a good approximation for the following development.

In order to minimize J, the Hamiltonian is formed by the maximum principle:

$$H = \lambda_1 g_1 + \lambda_2 g_2 + \lambda_3 g_3 + \lambda_4 g_4 - fQ_{aux}^2 - C(T_e - T_{set})^2$$

where λ_1–λ_4 are adjoint variables. A necessary condition for the Hamiltonian to be minimized with respect to the control is:

$$\frac{\partial H}{\partial Q_{aux}} = 0 = -2fQ_{aux} + \frac{\lambda_3}{C_f} \Rightarrow Q_{aux} = \frac{\lambda_3}{2fC_f} \tag{15}$$

The optimal control depends on $\lambda_3(t)$ and $f(t)$. The equations defining the adjoint variables are:

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial T_s} = (a_2 + a_3)\lambda_1 - a_5\lambda_2 \tag{16}$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial T_w} = -a_3\lambda_1 + (a_5 + a_6)\lambda_2 - a_{10}\lambda_4 \tag{17}$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial T_f} = -(a_8 + a_9)\lambda_3 - (a_{12} - a_{14})\lambda_4 \tag{18}$$

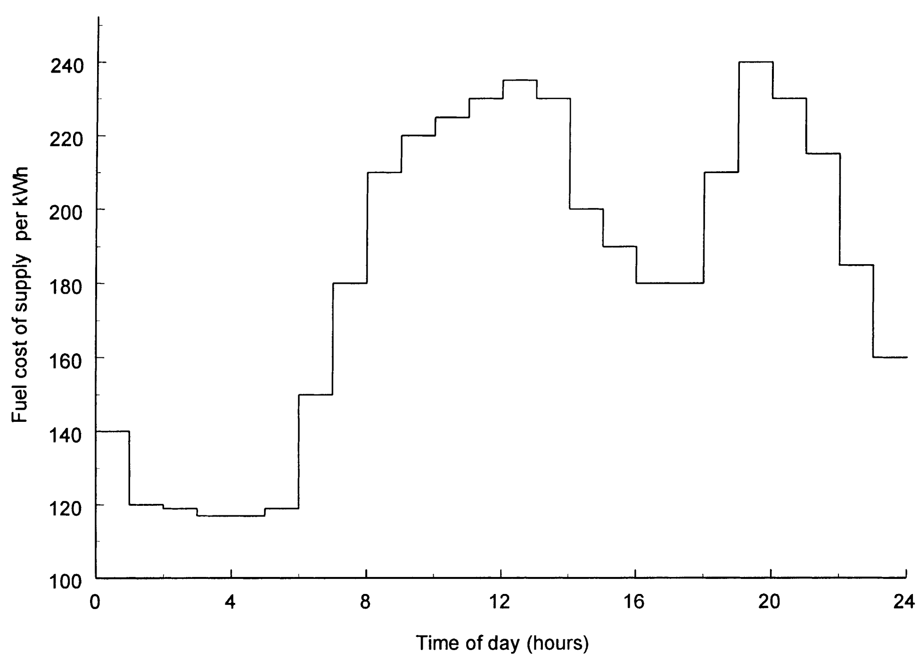

Fig. 2. Fuel cost of supply on utility peak day (April 1990).

$$\frac{d\lambda_4}{dt} = -\frac{\partial H}{\partial T_e} = -(a_7 + a_6)\lambda_2 - a_8\lambda_3 + (a_{10} + a_{11} + a_{12} + a_{13})\lambda_4 + 2C(T_e - T_{set}) \quad (19)$$

Once $\lambda_3(t)$ is finally found by solving the above non-omogenic system of differential equations [7], it is substituted into Eq. (15) to determine the optimal control for the entire period of the optimization.

4. Results and conclusions

An example of a passive-solar-heated residence in Xanthi, Greece will be described and the dynamic optimization technique, based on the optimum exploitation of off-peak

Table 1
Parameter values

A_{glaz} (m^2)	15 (160 ft^2)
A_f (m^2)	120 (1295 ft^2)
A_w (m^2)	16 (173 ft^2)
C_s (kJ/°C)	5000 (2632 Btu/°F)
C_w (kJ/°C)	9500 (5000 Btu/°F)
C_f (kJ/°C)	38000 (20000 Btu/°F)
C_e (kJ/°C)	19000 (10000 Btu/°F)
U_{sa} (W/m^2 °C)	10 (1.76 Btu/h ft^2 °F)
U_{sw} (W/m^2 °C)	8.5 (1.5 Btu/h ft^2 °F)
U_{we} (W/m^2 °C)	8.5 (1.5 Btu/h ft^2 °F)
U_{fe} (W/m^2 °C)	8.52 (1.56 Btu/h ft^2 °F)
U_{fg} (W/m^2 °C)	0.57 (0.1 Btu/h ft^2 °F)
m (kg/h)	50
C_p (kJ/kg °C)	0.83
UA_e (W/°C)	204 (388 Btu/h °F)

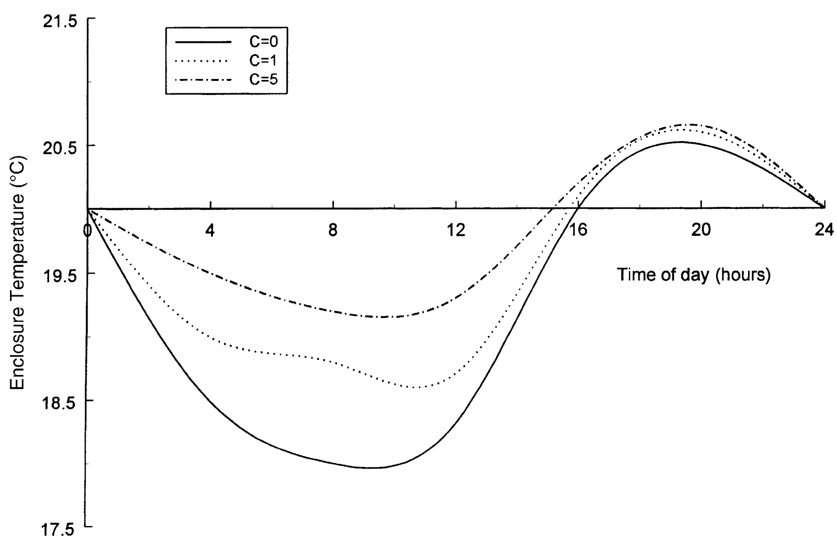

Fig. 3. Enclosure temperature response to optimal control for sunny day.

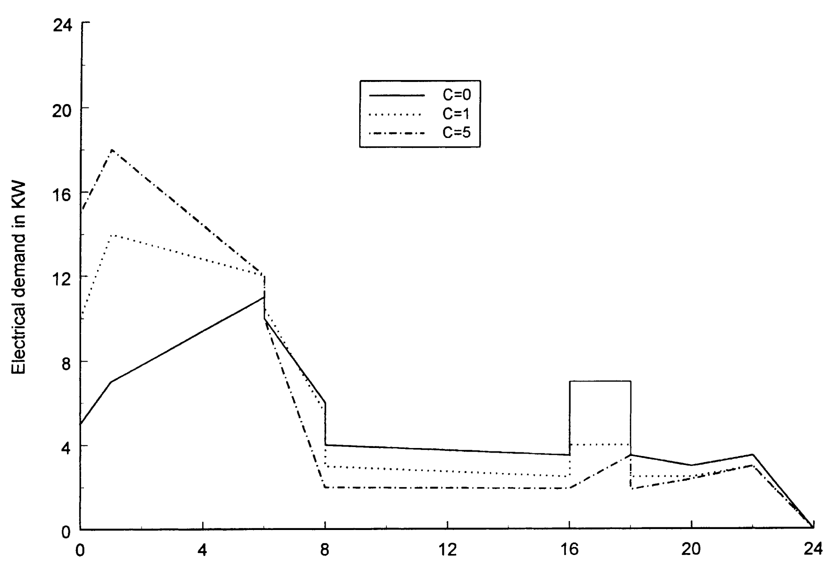

Fig. 4. Optimal control strategy for sunny day.

electricity for floor heating and the comfort weighting factor, will be applied. For this application, the passive heating system parameters are given in Table 1.

Fig. 2 shows a typical utility fuel costs, f as a function of time (on a peak day of a specific month—data taken from the Public Power Corporation—electrical utility in Greece for April 1990) because they vary with the overall utility demand at any time. Taking into consideration the values of f, the optimal control strategy for different weighting factor C is shown in Fig. 3. The corresponding enclosure temperature variations are in Fig. 4 for a typical sunny day.

On the sunny day, the enclosure temperature is kept closer to the desired temperature as the comfort weighting factor C is increased. When the comfort weighting factor is high, large instantaneous values of Q_{aux} can occur and when the factor is low, Q_{aux} has less variations throughout the day. It can be noticed that for the control strategy the estimate of temperature and solar radiation for the next day is very important. This is because future knowledge of the weather features, produces rather precise temperature control in the enclosure. One method to overcome this problem is the use of a direct input from a forecaster. The forecasted high temperature, low temperature and cloudiness are sufficient to give a reasonable estimate of the net heating load to be encountered during the next day.

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