Hence

$$h = \frac{q_s''}{T_s - T_m} = \frac{-2C_2\,(k/r_o)}{-C_2/2} = \frac{4k}{r_o}$$

and

$$Nu_D = \frac{hD}{k} = \frac{(4k/r_o) \times 2r_o}{k} = 8$$ ◁

8.3 The Energy Balance

8.3.1 General Considerations

Because the flow in a tube is completely enclosed, an energy balance may be applied to determine how the mean temperature $T_m(x)$ varies with position along the tube and how the total convection heat transfer q_{conv} is related to the difference in temperatures at the tube inlet and outlet. Consider the tube flow of Figure 8.6. Fluid moves at a constant flow rate $\dot{m}$, and convection heat transfer occurs at the inner surface. Typically, fluid kinetic and potential energy changes, as well as energy transfer by conduction in the axial direction, are negligible. Hence if no shaft work is done by the fluid as it moves through the tube, the only significant effects will be those associated with *thermal energy changes* and with *flow work.* Flow work is performed to move fluid through a control surface [5, 6] and, per unit mass of fluid, may be expressed as the product of the fluid pressure p and specific volume v ($v = 1/\rho$).

Applying conservation of energy, Equation 1.11a, to the differential control volume of Figure 8.6 and recalling the definition of the mean temperature, Equation 8.25, we obtain

$$dq_{\text{conv}} + \dot{m}(c_v T_m + pv) - \left[\dot{m}(c_v T_m + pv) + \dot{m}\frac{d\,(c_v T_m + pv)}{dx}\,dx\right] = 0$$

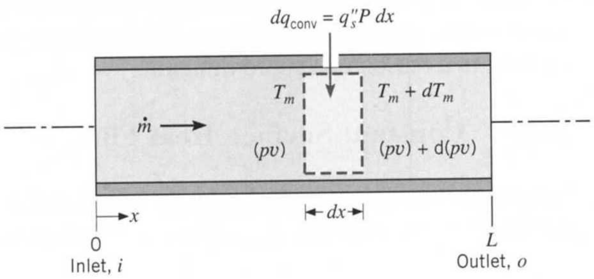

FIGURE 8.6 Control volume for internal flow in a tube.

or

$$dq_{\text{conv}} = \dot{m}d\,(c_v T_m + pv) \qquad (8.3$$

That is, the rate of convection heat transfer to the fluid must equal the rate at wh the fluid thermal energy increases plus the net rate at which work is done in mov the fluid through the control volume. If the fluid is assumed to be an *ideal g* ($pv = RT_m$, $c_p = c_v + R$) and c_p is assumed to be constant, Equation 8.35 reduces t

$$dq_{\text{conv}} = \dot{m}c_p\,dT_m \qquad (8.3$$

This expression may also be used to a good approximation for *incompressible li uids*. In this case $c_v = c_p$, and since v is very small, $d(pv)$ is generally much le than $d(c_v T_m)$.[1] Accordingly, Equation 8.36 again follows from Equation 8.35.

A special form of Equation 8.36 relates to conditions for the *entire* tube. I particular, integrating from the tube inlet i to the outlet o, it follows that

$$q_{\text{conv}} = \dot{m}c_p\,(T_{m,o} - T_{m,i}) \qquad (8.37)$$

where q_{conv} is the total tube heat transfer rate. This simple overall energy bal ance relates three important thermal variables (q_{conv}, $T_{m,o}$, $T_{m,i}$). *It is a genera expression that applies irrespective of the nature of the surface thermal or tub flow conditions.*

Equation 8.36 may be cast in a convenient form by expressing the rate o convection heat transfer to the differential element as $dq_{\text{conv}} = q_s''P\,dx$, where P is the surface perimeter ($P = \pi D$ for a circular tube). Substituting from Equa tion 8.28, it follows that

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p}h(T_s - T_m) \qquad (8.38)$$

This expression is an extremely useful result, from which the axial variation of T_m may be determined. If $T_s > T_m$, heat is transferred to the fluid and T_m increases with x; if $T_s < T_m$, the opposite is true.

The manner in which quantities on the right-hand side of Equation 8.38 vary with x should be noted. Although P may vary with x, most commonly it is a constant (a tube of constant cross-sectional area). Hence the quantity ($P/\dot{m}c_p$) is a constant. In the fully developed region, the convection coefficient h is also constant, although it varies with x in the entrance region (Figure 8.5). Finally, although T_s may be constant, T_m must always vary with x (except for the trivial case of no heat transfer, $T_s = T_m$).

The solution to Equation 8.38 for $T_m(x)$ depends on the surface thermal con dition. Recall that the two special cases of interest are *constant surface heat flux* and *constant surface temperature*. It is common to find one of these conditions existing to a reasonable approximation.

8.3.2 Constant Surface Heat Flux

For constant surface heat flux we first note that it is a simple matter to determine the total heat transfer rate q_{conv}. Since q_s'' is independent of x, it follows that

[1]The only exception arises when the pressure gradient is extremely large. This situation occurs when $\dot{m}$ is very large and/or A_c is very small (see Problem 8.10).